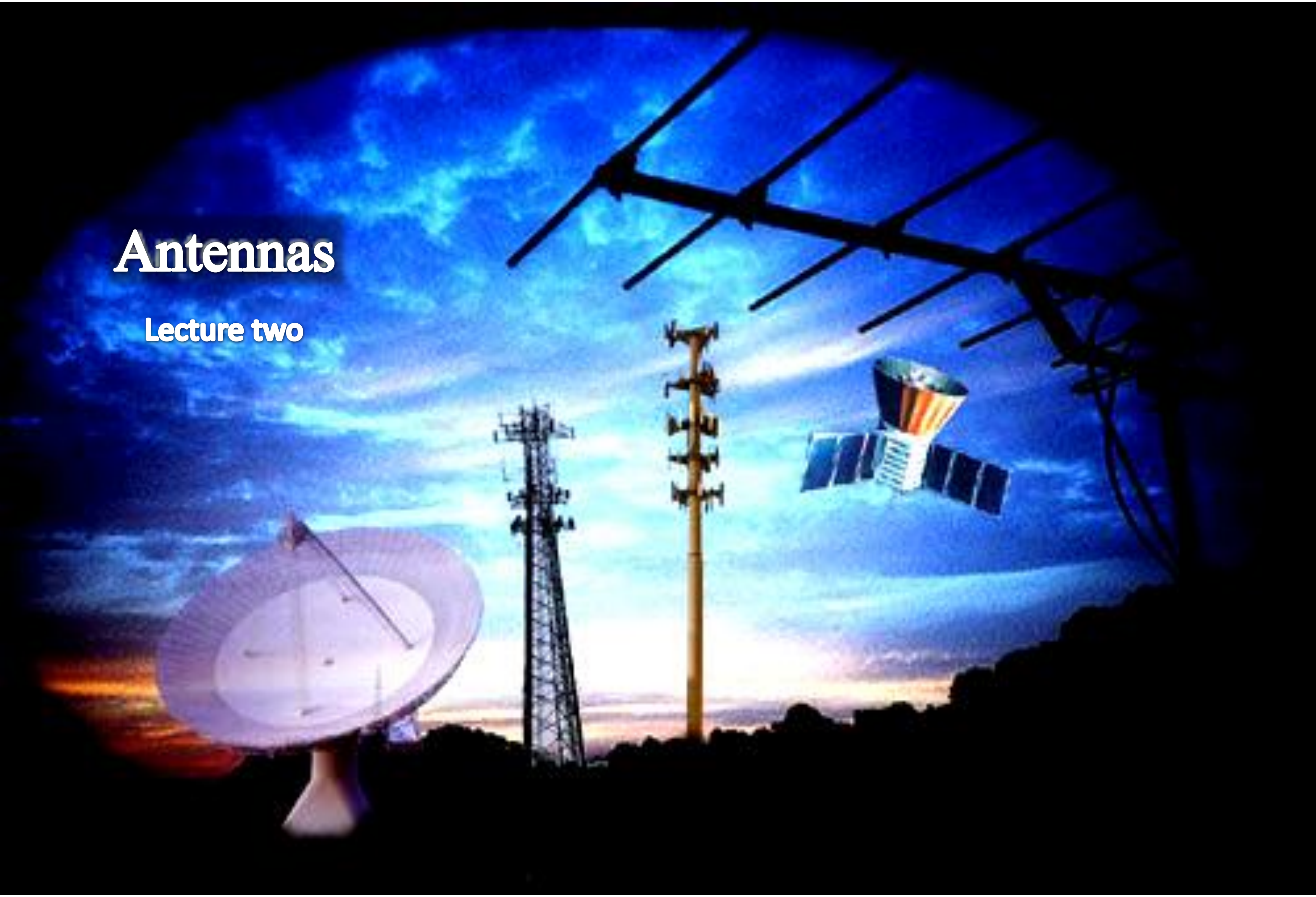
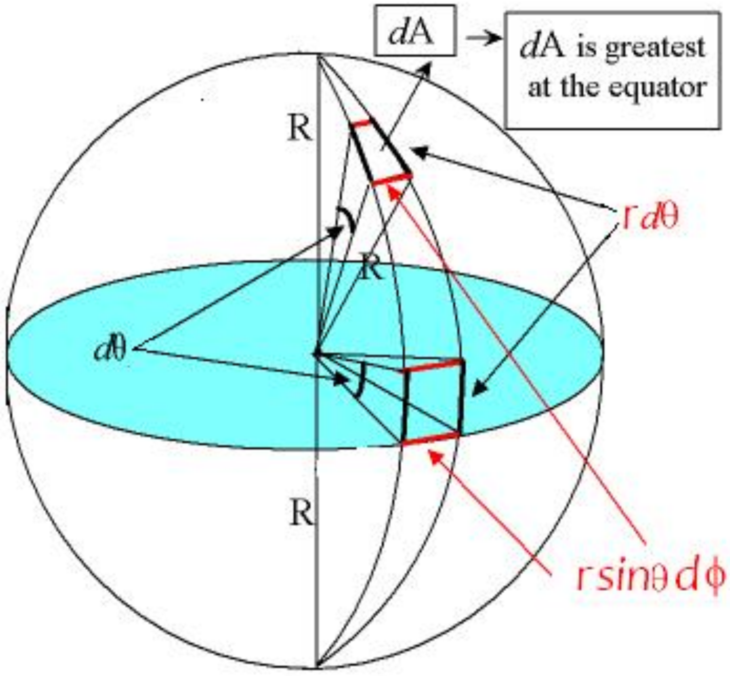
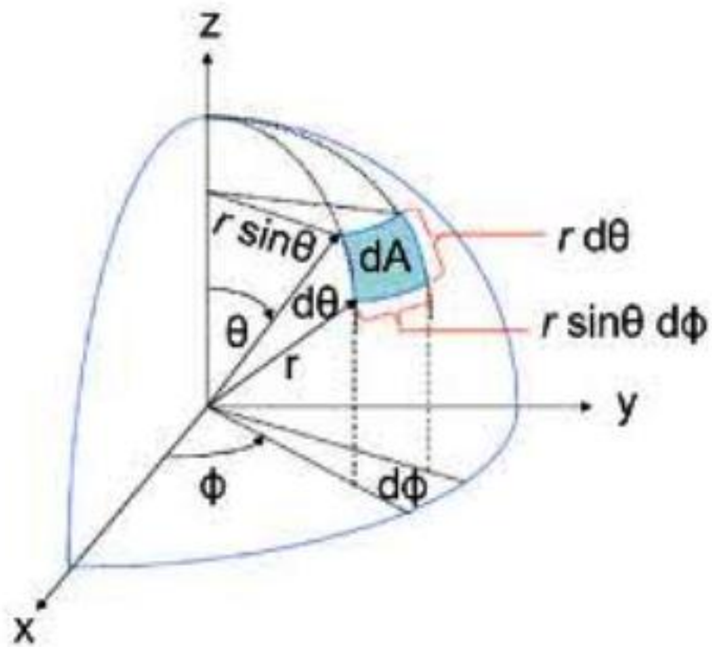

Antennas
Lecture two

Spherical coordinates

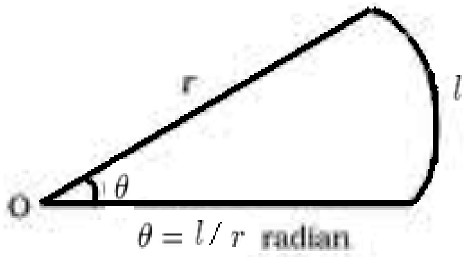
Infinitesimal area on a sphere of radius r





$$dA = r^2 \sin\theta \, d\theta \, d\phi$$

Plane angle and Solid Angle



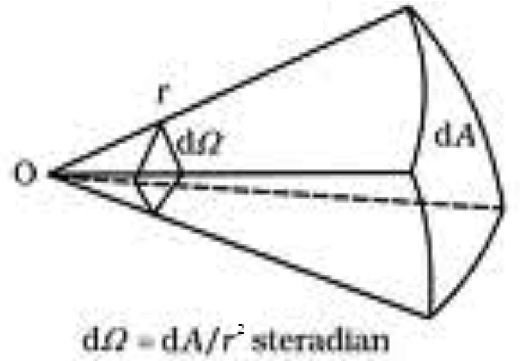
In two dimensions, the angle in radians is related to the arc length it cuts out:

$$\theta = \frac{l}{r}$$

where

l is arc length, and

r is the radius of the circle.



in three dimensions, the solid angle in steradians is relatec

area it cuts out:

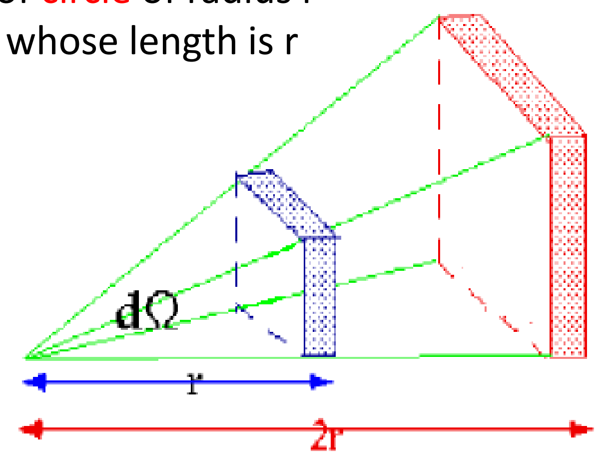
$$\Omega = \frac{S}{r^2}$$

where

S is the surface area of the spherical cap, $2\pi rh$, and

r is the radius of the sphere.

One radian defined as plane angle with
Its vertex at center of circle of radius r
And subtend an arc whose length is r



One steradian defined as solid angle with
Its vertex at center of sphere of radius r
And subtend by spherical surface area
Equal to r^2

-Number of steradian in sphere=4π

-since $dA = r^2 \sin\theta . d\theta . d\phi$

so $d\Omega = dA/r^2 = \sin\theta . d\theta . d\phi$

Fundamental Parameters of Antennas

1- Radiation Pattern 2- Beam-width 3- Radiation Power Density 4- Radiation Intensity 5- Directivity 6- Antenna Efficiency and Gain 7- Polarization

---

Some of antenna design factors are the strength of the radiated fields in different directions(radiation pattern), total power radiated compared to driven power(radiation efficiency), the antenna impedance to be matched to T.L. from feed, bandwidth.

1- Radiation Pattern

An antenna ***radiation pattern*** *is defined as a mathematical function* or a graphical representation of the radiation properties of the antenna as a function of space coordinates.

•Radiation patterns are conveniently represented in spherical coordinates.

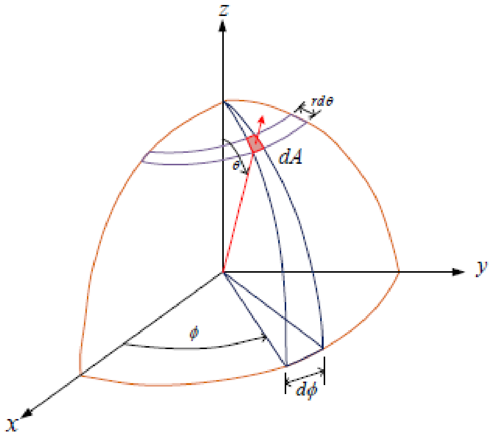
•Defined for the **Far Field Region**

It is drawn as:

•***Field patterns : Normalized*** $|E|$ $|H|$

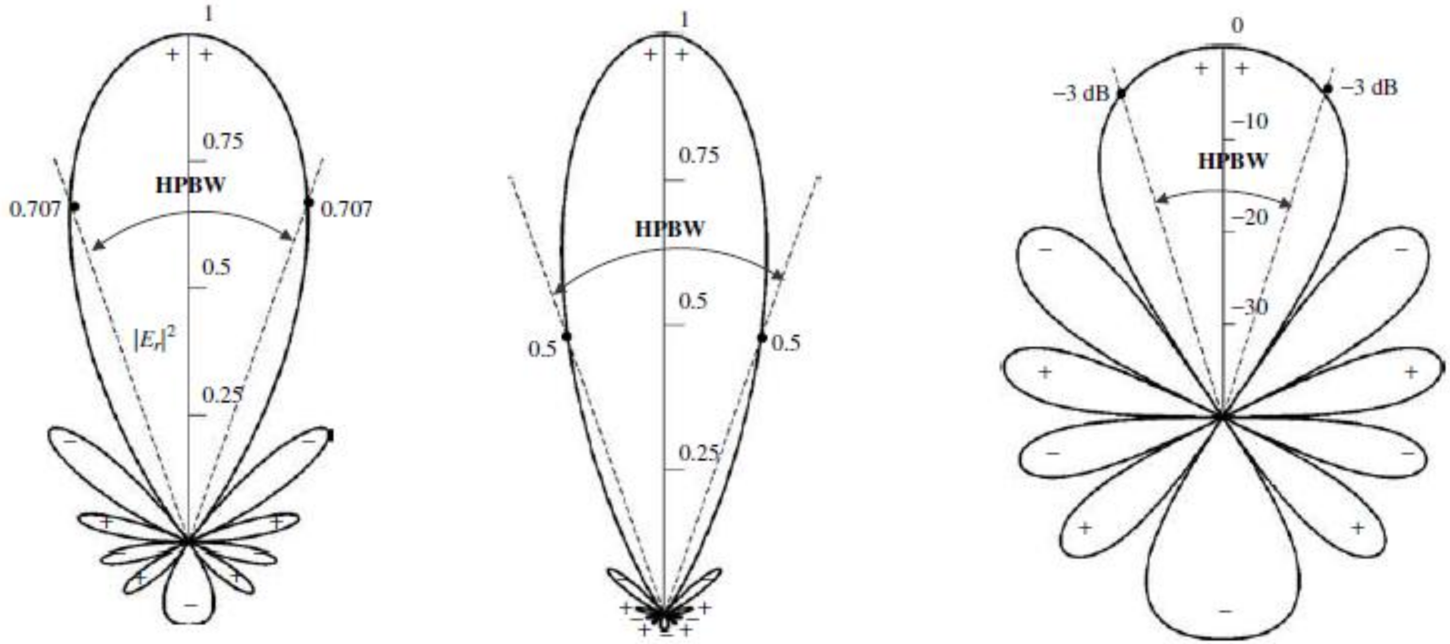
•***Or Power patterns: Normalized power*** $|E|^2$ ***vs. spherical coordinate position.***

(normalized with respect to their maximum value).



$$dA = r^2 \sin\theta \, d\theta \, d\phi.$$

Azimuth: ϕ Elevation: $\pi/2-\theta$.



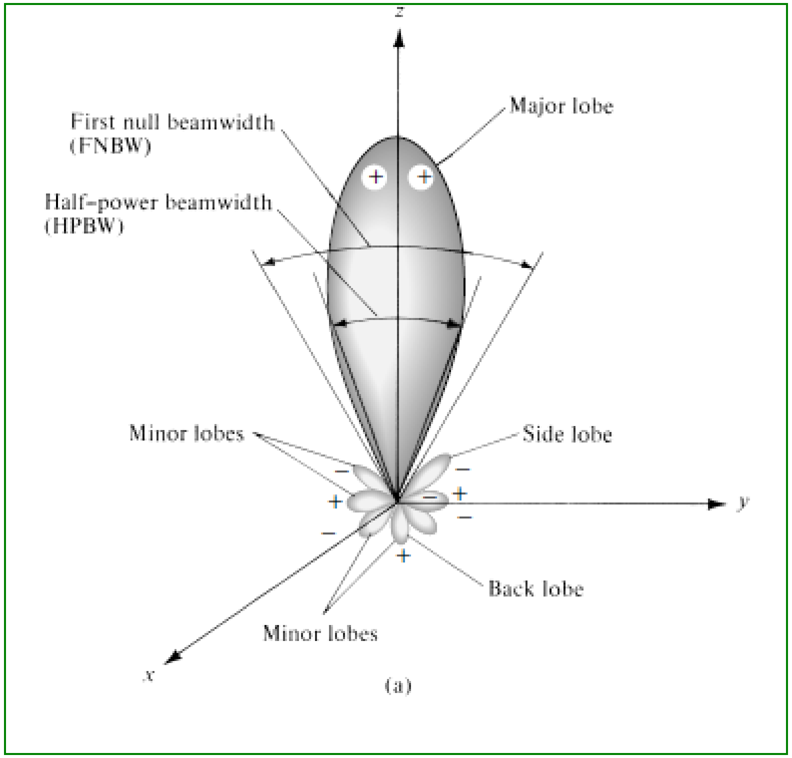
(a) Field pattern (in linear scale) (b) Power pattern (in linear scale) (c) Power pattern (in dB)

Figure 2.2 Two-dimensional normalized *field* pattern (*linear scale*), *power* pattern (*linear scale*), and *power* pattern (*in dB*) of a 10-element linear array with a spacing of $d = 0.25\lambda$.

All three patterns yield the same angular separation between the two half power points, ± 38.64, referred to as HPBW

HPBW is the angle between two directions having radiation intensity equal to one half of the beam maximum (measured at plane contains beam maximum)

FNBW is the angle separated between first nulls in the patterns.



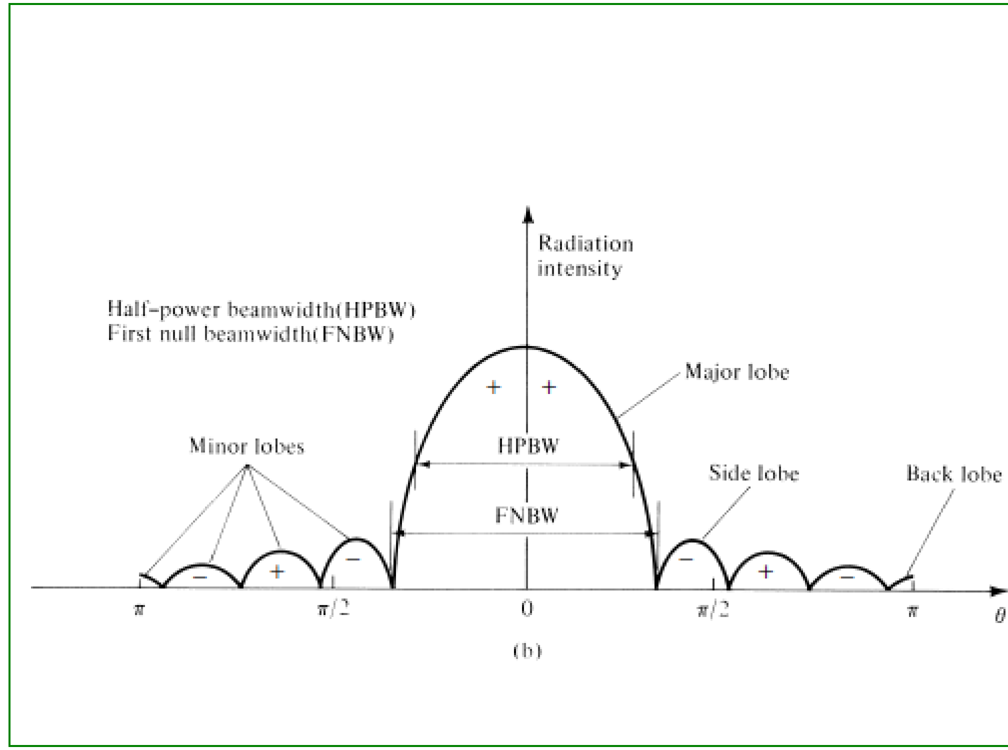


Figure 2.3 (a) Radiation lobes and beamwidths of an antenna pattern. (b) Linear plot of power pattern and its associated lobes and beamwidths.

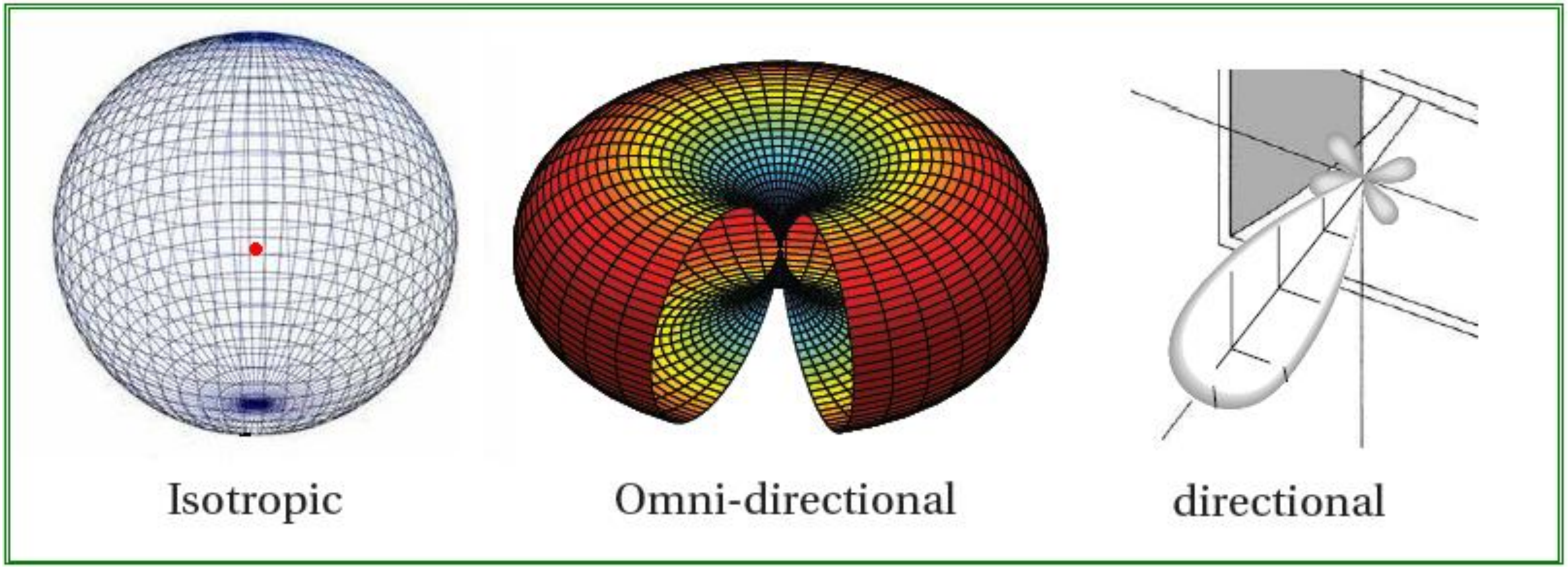
•In most radar systems, low side lobe ratios are very important to minimize false target indications through the side lobes.

Minor Lobe - any radiation lobe other than the major lobe. usually represent radiation in undesired directions, and they should be minimized.

Side Lobe - lobes, radiates in unwanted directions. Side lobes are normally the largest of the minor lobes.

Back Lobe - the radiation lobe opposite to the main lobe.(occupy hemisphere in direction opposite to major lobe)

- **Isotropic, Directional, and Omni-directional Patterns**
- **Definition 1 (Isotropic Radiator).** A hypothetical lossless antenna having equal radiation in all directions.
- **Definition 2 (Omni-directional Radiator).** An antenna having an essentially non directional pattern in a given plane (e.g., in azimuth) and a directional pattern in any orthogonal plane.
- **Definition 3 (Directional Radiator).** An antenna having the property of radiating or receiving more effectively in some directions than in others.



Example 1

For Infinitesimal Dipole

$$\mathbf{W}_{av} = \frac{1}{2}\mathrm{Re}(\mathbf{E}\times\mathbf{H}^*) = \hat{\mathbf{a}}_r \frac{1}{2\eta}|E_\theta|^2 = \hat{\mathbf{a}}_r \frac{\eta}{2}\left|\frac{kI_0 l}{4\pi}\right|^2 \frac{\sin^2\theta}{r^2}$$

$$U = r^2\, W_{av} = \frac{\eta}{2}\left(\frac{kI_0 l}{4\pi}\right)^2 \sin^2\theta = \frac{r^2}{2\eta}|E_\theta(r,\theta,\phi)|^2$$

$$U_{max} = \frac{\eta}{2}\left(\frac{kI_0 l}{4\pi}\right)^2$$

Radiation pattern $U/U_{max} = \sin^2\theta$

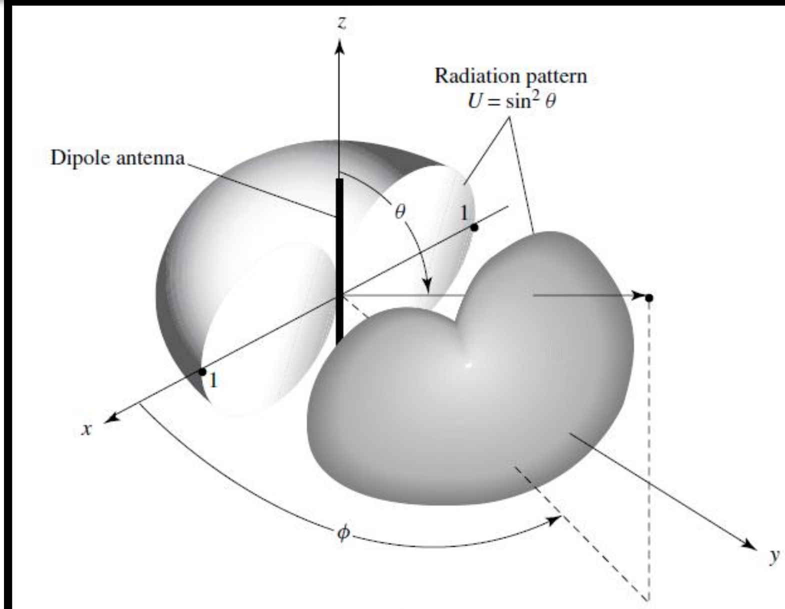
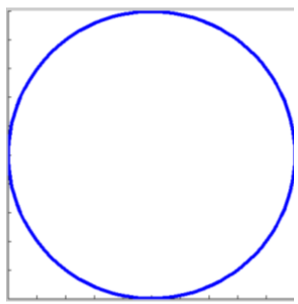


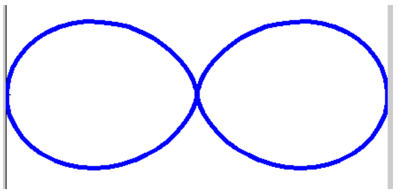
Figure 4.3 Three-dimensional radiation pattern of infinitesimal dipole.

• To Draw at X-Y plane ($\theta = \pi/2$, $\varphi = 0 - 2\pi$)
Radiation pattern $= \sin^2(\pi/2) = 1$ for all φ values.

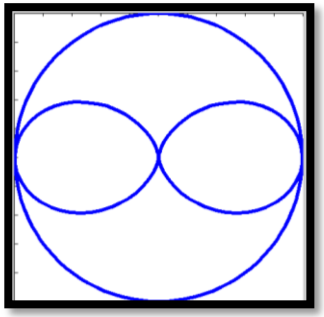


• To draw at X-Z plane ($\varphi=0$, $\theta=0-\pi$ then $\varphi=180$, $\theta=0-\pi$)

θ	0	30	45	60	90	120	135	150	180
$\sin^2\theta$	0	.25	.5	.75	1	.75	.5	.25	0



HPBW=90^0
FNBW=180^0



Example 2

Example 2.4

The normalized radiation intensity of an antenna is represented by

$$U(\theta) = \cos^2(\theta)\cos^2(3\theta), \quad (0 \le \theta \le 90^\circ, \quad 0^\circ \le \phi \le 360^\circ)$$

The three- and two-dimensional plots of this, plotted in a linear scale, are shown in Figure 2.11. Find the

a. half-power beamwidth HPBW (*in radians and degrees*)

b. first-null beamwidth FNBW (*in radians and degrees*)

$\cos\theta_h \ .\cos 3\theta_h = \sqrt{.5}$

$.5\,(\cos 4\theta_h + \cos 2\theta_h) = .707$

$.5\,(2\cos^2 2\theta_h - 1 + \cos 2\theta_h) = .707$

LET $\cos 2\theta_h = X$ solve equation THEN $X = .876$ so $\cos 2\theta_h = .876$

$2\theta_h = 28.74^0$

HPBW $= 28.74^0 = .5$ radians

Same for nulls $.5\,(2\cos^2 2\theta_n - 1 + \cos 2\theta_n) = 0$

$\cos^2 2\theta_n - .5 + .5\cos 2\theta_n = 0$ solve $2\theta_n = 60^0$ or 180^0 take the smallest for first null. FNBW $= 60^0 = \pi/3$ radians

Components in the Amplitude Pattern

- There would be, the electric-field components (E_θ, E_ϕ) at each observation point on the surface of a sphere of constant radius.
- In the far field, the radial E_r component for all antennas is zero or vanishingly small.
- Some antennas, depending on their geometry and also observation distance, may have only one, or two components.
- In general, the magnitude of the total electric field would be $|E| = \sqrt{|E_\theta|^2 + |E_\phi|^2}$

- For Isotropic radiator which radiates equally in all directions(not exist but used as reference to compared with other antenna), the power density equal to

$$\mathbf{W}_0 = \hat{\mathbf{a}}_r W_0 = \hat{\mathbf{a}}_r \left(\frac{P_{\text{rad}}}{4\pi r^2}\right) \quad (\text{W/m}^2)$$

$W_{rad} \Rightarrow$ watt/m^2

$U \Rightarrow$ watt/solid angle = watt/m^2/r^2

$U = r^2 W_{rad}$

Radiation intensity: defined as

The power radiated from antenna per unit solid angle U(ɵ,ϕ)

$$U(\theta, \phi) = \frac{r^2}{2\eta}|\mathbf{E}(r, \theta, \phi)|^2 \simeq \frac{r^2}{2\eta}\left[|E_\theta(r, \theta, \phi)|^2 + |E_\phi(r, \theta, \phi)|^2\right] \quad (2\text{-}12a)$$

where

$\mathbf{E}(r, \theta, \phi)$ = far-zone electric-field intensity of the antenna

E_θ, E_ϕ = far-zone electric-field components of the antenna

η = intrinsic impedance of the medium

U = radiation intensity (W/unit solid angle)

The total power is obtained by integrating the radiation intensity, as given by (2-12), over the entire solid angle of 4π. Thus

$$P_{\text{rad}} = \oiint_{\Omega} U\, d\Omega = \int_0^{2\pi} \int_0^{\pi} U \sin\theta\, d\theta\, d\phi \quad (2\text{-}13)$$

radiation intensity of an isotropic source $U_0 = \dfrac{P_{\text{rad}}}{4\pi}$

Directivity

is the ratio of radiation intensity in a given direction to isotropic radiation intensity

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{\text{rad}}}$$

If the direction is not specified → (maximum directivity)

$$D_{\max} = D_0 = \frac{U|_{\max}}{U_0} = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{\text{rad}}}$$

Example 2.5

find the maximum directivity of the antenna whose radiation intensity is $U = A_0 \sin\theta$ Write an expression for the directivity as a function of the directional angles θ and ϕ.

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi = A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta \, d\phi = \pi^2 A_0$$

Using (2-16a), we find that the maximum directivity is equal to

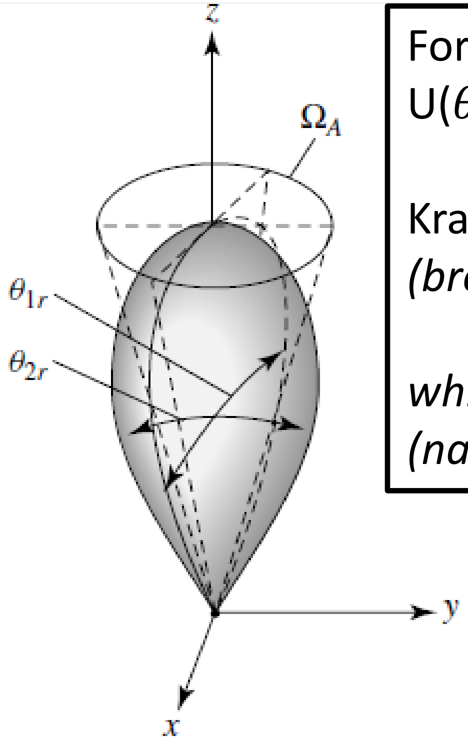
$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4}{\pi} = 1.27$$

Since the radiation intensity is only a function of θ, the directivity as a function of the directional angles is represented by

$$D = D_0 \sin\theta = 1.27 \sin\theta$$

Beam solid Angle:

For antennas with one narrow major lobe and very negligible minor lobes, the beam solid angle is approximately equal to the product of the half-power beam widths in two perpendicular planes



For radiation intensity
$U(\theta,\phi) = B_0 \cos^n(\theta)$ at $0<\theta<\pi/2$, $0<\phi<2\pi$ and 0 elsewhere

Kraus' formula is more accurate for small values of *n* *(broader patterns)* *(let us take it as n* <10 according to table 2.1)

while Tai & Pereira's is more accurate for large values of *n* *(narrower patterns).*

Beam solid angles $\Omega_A = \theta_{1r} \cdot \theta_{2r}$

$$D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\theta_{1r} \cdot \theta_{2r}} \quad \text{(Kraus)} \quad \text{or} \quad D_0 \simeq \frac{32 \ln 2}{\theta_{1r}^2 + \theta_{2r}^2} \quad \text{(Tai-Pereira)}$$

θ_{1r} = half-power beamwidth in one plane (rad)

θ_{2r} = half-power beamwidth in a plane at a right angle to the other (rad)

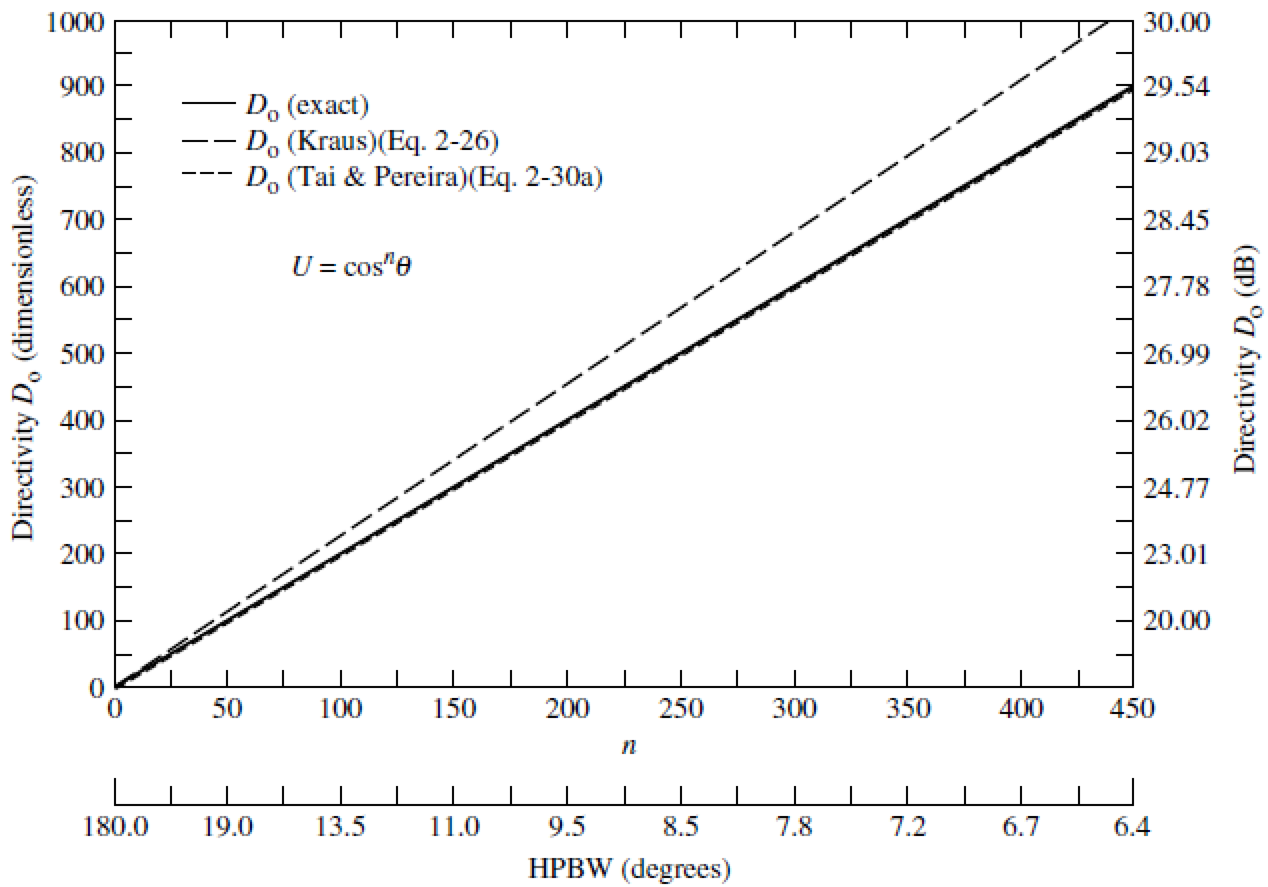


Figure 2.16 Comparison of exact and approximate values of directivity for directional $U = \cos^n\theta$ power patterns.

Example 2.7

The radiation intensity of the major lobe of many antennas can be adequately represented by

$$U = B_0 \cos\theta$$

where B_0 is the maximum radiation intensity. The radiation intensity exists only in the upper hemisphere ($0 \le \theta \le \pi/2$, $0 \le \phi \le 2\pi$), and it is shown in Figure 2.15.

Find the

a. beam solid angle; exact and approximate.

b. maximum directivity; exact using (2-23) and approximate using (2-26).

b. *Directivity* D_0:

Exact: $D_0 = \dfrac{4\pi}{\Omega_A} = \dfrac{4\pi}{\pi} = 4$ (dimensionless) $= 6.02$ dB

The same exact answer is obtained using (2-16a).

Approximate: $D_0 \approx \dfrac{4\pi}{\Omega_A} = \dfrac{4\pi}{4.386} = 2.865$ (dimensionless) $= 4.57$ dB

The exact maximum directivity is 4 and its approximate value, using (2-26), is 2.865. Better approximations can be obtained if the patterns have much narrower beamwidths

Example 2.8

Design an antenna with omnidirectional amplitude pattern with a half-power beamwidth of 90°. Express its radiation intensity by $U = \sin^n \theta$. Determine the value of n, max directivity

Solution: Since the half-power beamwidth is 90°, the angle at which the half-power point occurs is $\theta = 45°$. Thus

$$U(\theta = 45^\circ) = 0.5 = \sin^n(45^\circ) = (0.707)^n$$

or

$$n = 2$$

Therefore, the radiation intensity of the omnidirectional antenna is represented by $U = \sin^2 \theta$. An infinitesimal dipole (see Chapter 4) or a small circular loop (see Chapter 5) are two antennas which possess such a pattern.

Using the definition of (2-16a), the exact directivity is

$$U_{max} = 1$$

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} \sin^2\theta \, \sin\theta \, d\theta \, d\phi = \frac{8\pi}{3}$$

$$D_0 = \frac{4\pi}{8\pi/3} = \frac{3}{2} = 1.761 \text{ dB}$$

$$\int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3}$$

Antenna efficiency

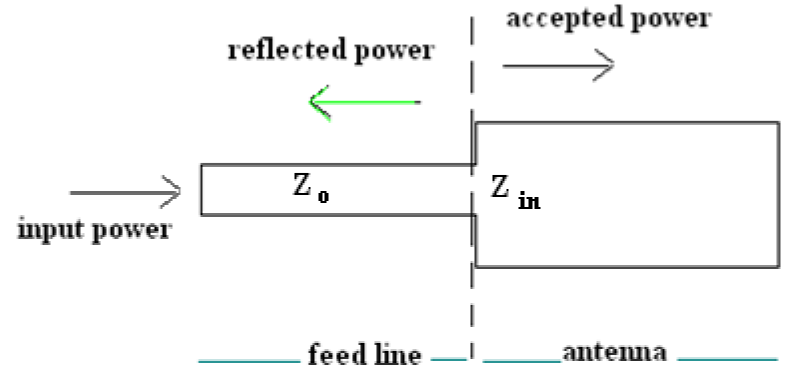
$$e_t = e_{cd} * e_r$$

Where: $e_t = P_{rad}/P_{input}$ total efficiency

$e_{cd} = P_{rad}/P_{accept}$ radiation efficiency

Contains conduction and dielectric losses

$e_r = 1 - |\Gamma|^2$ Mismatch loss



$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0},$$

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}.$$

Γ = voltage reflection coefficient at the input terminals of the antenna Z_{in} = antenna input impedance, Z_0 = characteristic impedance of the transmission line. VSWR = voltage standing wave ratio

efficiency is very close to:

• 100% (or 0 dB) for dish, horn antennas,
or half-wavelength dipoles with no lossy materials around them

•20%- 70% for Mobile antenna(microstrip) ,losses due to material surround antenna, and dielectric losses

Gain

- Defined as ratio of radiation intensity in a given direction to radiation intensity obtained if accepted power where radiated isotropic.
- Gain does not account for losses arising from impedance mismatches

$$G(\theta, \phi) = e_{cd} \left[4\pi \frac{U(\theta, \phi)}{P_{\text{rad}}} \right]$$

$$G(\theta, \phi) = e_{cd} D(\theta, \phi)$$

the maximum value of the gain

$$G_0 = G(\theta, \phi)|_{\max} = e_{cd} D(\theta, \phi)|_{\max} = e_{cd} D_0$$

Absolute Gain

Take into account losses arising from impedance mismatches

$$G_{\text{abs}} = e_r G(\theta, \phi) = (1 - |\Gamma|^2) G(\theta, \phi) = e_r e_{cd} D(\theta, \phi) = e_t D(\theta, \phi).$$

where

$e_r = (1 - |\Gamma|^2)$, reflection (mismatch) efficiency,

e_t = total efficiency

- For the maximum values

$$G_{0\text{abs}} = e_t D_0.$$

$$G_0(\text{dB}) = 10 \log_{10}[e_{cd} D_0 \text{ (dimensionless)}]$$

example

Example 2.10

A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by

$$U = B_0 \sin^3 \theta$$

find the maximum absolute gain of this antenna.

$$U|_{\max} = U_{\max} = B_0$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi = 2\pi B_0 \int_0^{\pi} \sin^4 \theta \, d\theta = B_0 \left(\frac{3\pi^2}{4} \right)$$

$$D_0 = 4\pi \frac{U_{\max}}{P_{\text{rad}}} = \frac{16}{3\pi} = 1.697$$

Since the antenna was stated to be lossless, then the radiation efficiency $e_{cd} = 1$.

Thus, the total maximum gain is equal to

$$G_0 = e_{cd} D_0 = 1(1.697) = 1.697$$

which is identical to the directivity because the antenna is lossless.

There is another loss factor which is not taken into account in the gain. That is the loss due to reflection or mismatch losses between the antenna (load) and the transmission line. This loss is accounted for by the reflection efficiency

$$e_t = (1 - |\Gamma|^2) = \left(1 - \left| \frac{73 - 50}{73 + 50} \right|^2 \right) = 0.965$$

$$e_t = e_r e_{cd} = 0.965$$

$$G_{0abs} = e_t D_0 = 0.965(1.697) = 1.6376$$

Bandwidth

defined as the range of frequencies where performance of antenna(antenna characteristic as input impedance ,pattern, polarization ,gain, radiation efficiency,…) conforms to specific standards (according to antenna application).

For narrowband antennas, the BW is expressed as a percentage of the frequency difference over the center frequency:

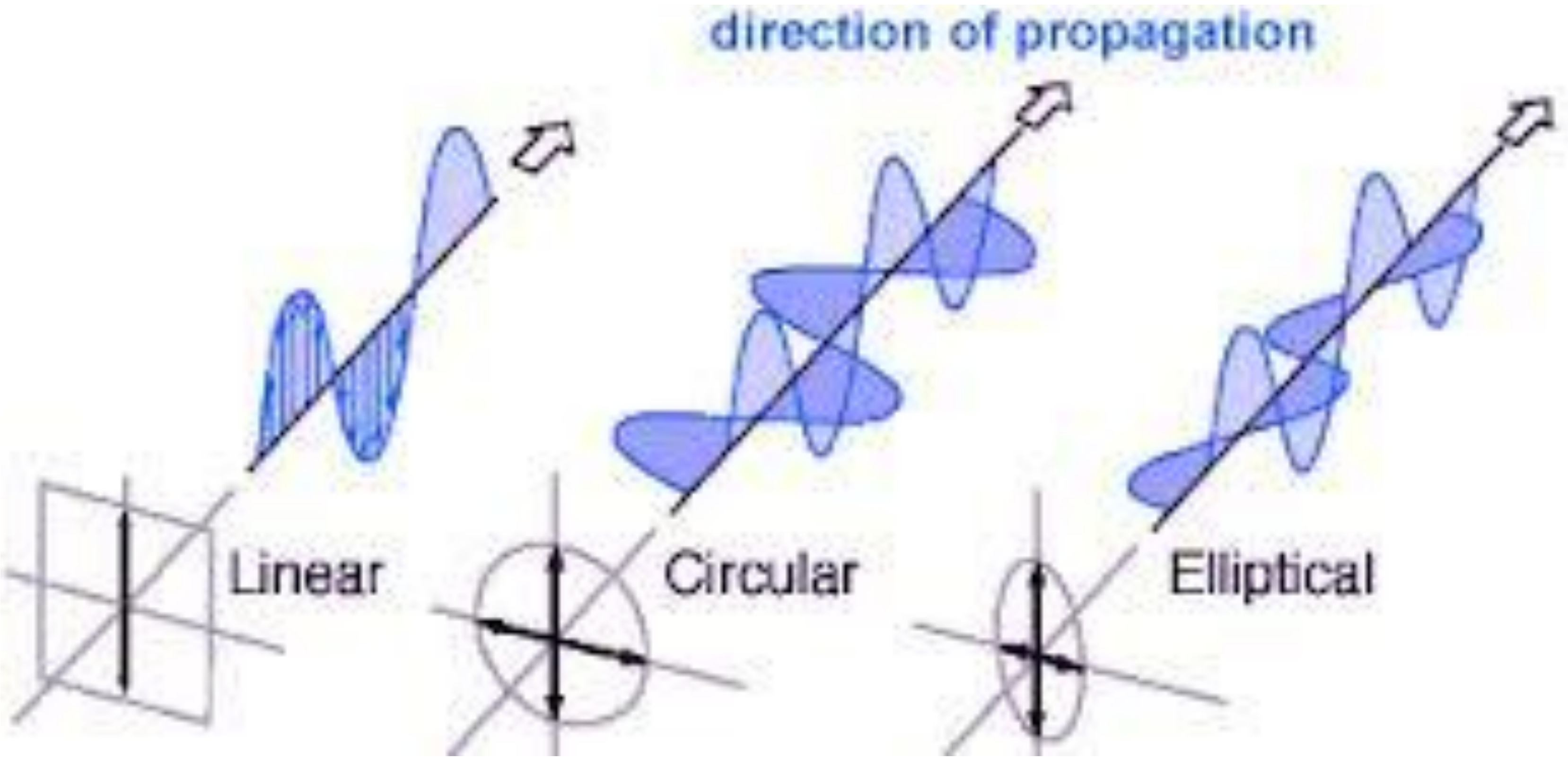
$$BW = \frac{f_{upper} - f_{lower}}{f_0} \cdot 100\ \%.$$

Usually, $f_0 = (f_{upper} + f_{lower}) / 2$

For broadband antennas $BW = f_{upper} / f_{lower}$

Broadband antennas with BW as large as 40:1 have been designed. Such antennas are referred to as *frequency independent antennas*.

Polarization
defined as trace of the radiated electric field vector (linear, circular, elliptical) along the direction of propagation.



2.12.1 Linear, Circular, and Elliptical Polarizations

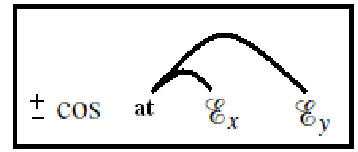
The instantaneous field of a plane wave, traveling in the negative z direction, can be written as

$$\mathscr{E}(z;t) = \hat{\mathbf{a}}_x \mathscr{E}_x(z;t) + \hat{\mathbf{a}}_y \mathscr{E}_y(z;t)$$

$$\mathscr{E}_x(z;t) = E_{xo} \cos(\omega t + kz + \phi_x)$$

$$\mathscr{E}_y(z;t) = E_{yo} \cos(\omega t + kz + \phi_y)$$

Linear Polarization $\qquad \phi_y - \phi_x = n\pi, \quad n = 0, 1, 2, 3, \ldots$

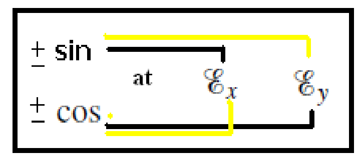


Circular Polarization

Circular polarization can be achieved *only* when the magnitudes of the two components are the same *and* the time-phase difference between them is odd multiples of $\pi/2$.

$$|\mathscr{E}_x| = |\mathscr{E}_y| \Rightarrow E_{xo} = E_{yo}$$

$$\phi_y - \phi_x = \begin{cases} +(\frac{1}{2} + 2n)\pi, n = 0, 1, 2, \ldots & \text{for CW} \\ -(\frac{1}{2} + 2n)\pi, n = 0, 1, 2, \ldots & \text{for CCW} \end{cases}$$



Elliptical Polarization

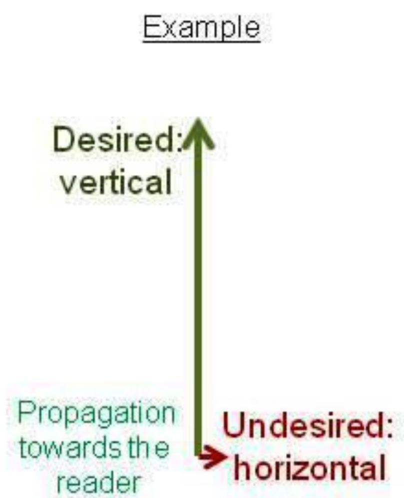
Elliptical polarization can be attained *only* when the time-phase difference between the two components is odd multiples of $\pi/2$ *and* their magnitudes are not the same *or* when the time-phase difference between the two components is not equal to multiples of $\pi/2$ (irrespective of their magnitudes). That is,

$$|\mathscr{E}_x| \neq |\mathscr{E}_y| \Rightarrow E_{xo} \neq E_{yo}$$

$$\phi_y - \phi_x = \begin{cases} +(\frac{1}{2} + 2n)\pi & \text{for CW} \\ -(\frac{1}{2} + 2n)\pi & \text{for CCW} \end{cases} \qquad n = 0, 1, 2, \ldots$$

Cross Polarization

Desired polarization	Undesired perpendicular polarization	Cross polarization [dB]
Vertical	Horizontal	Vertical – horizontal
Horizontal	Vertical	Horizontal – Vertical
RHCP	LHCP	RHCP – LHCP
LHCP	RHCP	LHCP – RHCP



Every antenna radiates in a desired polarization it was design to, but, in addition to that it has a "leakage" that radiates in the perpendicular polarization to the desired one. The ratio between the undesired polarization to the desired polarization is the ***cross polarization*** (in dB units it is a difference) .

In many cases, like in cellular base station antennas or in point to point antennas, the antennas are designed to have as low ***cross polarization*** as possible.